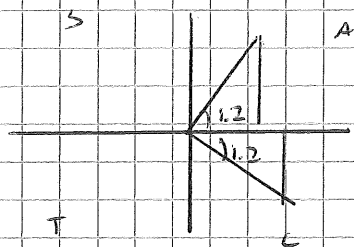


(1) a) $y = (3x+1)^5 \rightarrow \frac{dy}{dx} = 5 \times 3 (3x+1)^4$
 $= 15(3x+1)^4$

b) $y = \ln(3x+1) \rightarrow y = \ln(t) \quad t = 3x+1$
 $\frac{dy}{dt} = 1/t \quad \frac{dt}{dx} = 3$
 $\therefore \frac{dy}{dx} = 3/t = \frac{3}{3x+1}$

c) $y = (3x+1)^5 \ln(3x+1) \quad u = (3x+1)^5 \quad v = \ln(3x+1)$
 (Product Rule) $\frac{du}{dx} = 15(3x+1)^4 \quad \frac{dv}{dx} = \frac{3}{3x+1}$
 $\frac{dy}{dx} = 15(3x+1)^4 \ln(3x+1) + \frac{3(3x+1)^5}{3x+1}$
 $= 15(3x+1)^4 \ln(3x+1) + 3(3x+1)^4$

(2) a) $\sec(x) = 3 \rightarrow \cos(x) = 1/3$
 $x = 1.2309...$



$x = 1.23, 5.05$

b) $\tan^2(x) = 2 \sec(x) + 2 \quad \sec^2(x) = 1 + \tan^2(x)$
 $\sec^2(x) - 1 = 2 \sec(x) + 2$
 $\rightarrow \sec^2(x) - 2 \sec(x) - 3 = 0$

c) $(\sec(x) - 3)(\sec(x) + 1) = 0$

$\sec(x) = 3$

$x = 1.23, 5.05$

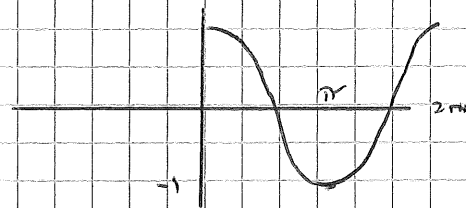
(from a)

$\sec(x) = -1$

$\rightarrow \cos(x) = -1$

$x = 3.14 (\pi)$

(from graph)



$x = 1.23, 3.14, 5.05$

(3) a) $y = x \cos(2x)$ $u = x$ $v = \cos(2x)$
 $\frac{du}{dx} = 1$ $\frac{dv}{dx} = -2 \sin(2x)$

$\frac{dy}{dx} = \cos(2x) - 2x \sin(2x)$

b) i) At A, $\frac{dy}{dx} = 0$

$\therefore \cos(2x) - 2x \sin(2x) = 0$

\therefore by $\cos(2x) \neq 0$ $1 - 2x \tan(2x) = 0$

at A, $x = \alpha$ ~~$2x \tan(2x) = 1$~~

$\rightarrow 1 - 2\alpha \tan(2\alpha) = 0$

ii) ~~Let~~ Let $f(\alpha) = 1 - 2\alpha \tan(2\alpha)$

$f(0.4) = 1 - 2(0.4) \tan(0.8) = 0.176\dots$

$f(0.5) = 1 - 2(0.5) \tan(1) = -0.557\dots$

change of sign, \therefore root lies between 0.4 & 0.5

iii) $1 - 2x \tan(2x) = 0$

$1 = 2x \tan(2x)$

$\frac{1}{2x} = \tan(2x)$

$\tan^{-1}(\frac{1}{2x}) = 2x$

$\rightarrow \alpha = \frac{1}{2} \tan^{-1}(\frac{1}{2\alpha})$

$\alpha = x!$

iv) $x_1 = 0.4$

$x_2 = \frac{1}{2} \tan^{-1}(\frac{1}{2 \times 0.4}) = 0.448\dots$

$x_3 = \frac{1}{2} \tan^{-1}(\frac{1}{2 \times 0.448}) = 0.4200\dots = 0.42$ (2sf)

c) $\int x \cos(2x) dx$

$u = x$
 $\frac{du}{dx} = 1$

$v = \cos(2x)$
 $\frac{dv}{dx} = -2 \sin(2x)$

$uv - \int v \frac{du}{dx}$

$= \frac{1}{2} x \sin(2x) - \int \frac{1}{2} \sin(2x)$

$= \left[\frac{1}{2} x \sin(2x) + \frac{1}{4} \cos(2x) \right]_0^{0.5}$

$= \frac{1}{2} (0.5) \sin(1) + \frac{1}{4} \cos(1) - 0 - \frac{1}{4} \cos(0)$

$= 0.095443\dots = 0.0954$ (3sf)

4) a) Range $f(x) = \geq 0$

b) i) let $y = \frac{1}{2x-3}$

OR

$$y(2x-3) = 1$$

$$2x-3 = \frac{1}{y}$$

$$2x = \frac{1}{y} + 3$$

$$x = \frac{1}{2y} + \frac{3}{2}$$

$$\Rightarrow y = \frac{1}{2x} + \frac{3}{2}$$

$$y(2x-3) = 1$$

$$2xy - 3y = 1$$

$$2xy = 1 + 3y$$

$$x = \frac{1+3y}{2y}$$

$$\Rightarrow y = \frac{1+3x}{2x}$$

ii) Range $g^{-1}(x) = \text{Domain } g(x) \rightarrow \neq \frac{3}{2}$

c) $f \circ g(x) = f\left(\frac{1}{2x-3}\right) = \left(\frac{1}{2x-3}\right)^2$

$$\left(\frac{1}{2x-3}\right)^2 = 9$$

$$\frac{1}{2x-3} = \pm 3$$

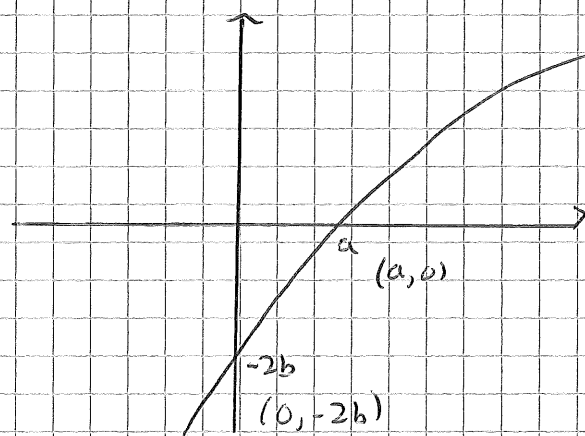
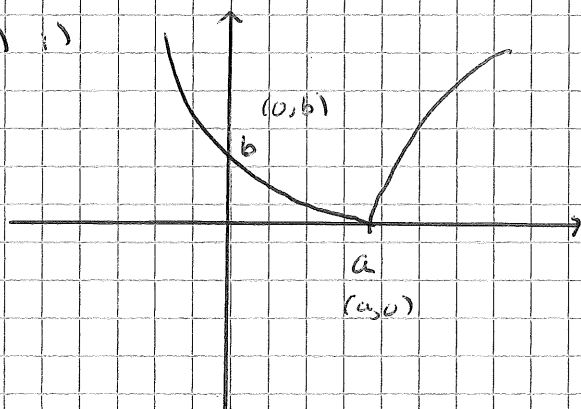
$$2x-3 = \pm \frac{1}{3}$$

$$2x = \pm \frac{1}{3} + 3$$

$$x = \frac{\pm \frac{1}{3} + 3}{2}$$

$$x = \frac{5}{3}, \frac{4}{3}$$

5) a) i)



b) i) $\boxed{+1}$ Translation $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$\boxed{4}$ Stretch, scale factor 4, parallel to y axis

$\boxed{-2}$ Translation $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$

ii) crosses y axis when $x = 0$

$$y = 4 \ln(1) - 2 = -2 \quad (0, -2)$$

crosses x axis when $y = 0$

$$0 = 4 \ln(x+1) - 2$$

$$2 = 4 \ln(x+1)$$

$$\frac{1}{2} = \ln(x+1)$$

$$e^{1/2} = x+1$$

$$x = e^{1/2} - 1 \quad (e^{1/2} - 1, 0)$$

6) a) $y = (e^{3x} + 1)^{1/2}$

$$y = t^{1/2}$$

$$t = e^{3x} + 1$$

$$\frac{dy}{dt} = \frac{1}{2} t^{-1/2}$$

$$\frac{dt}{dx} = 3e^{3x}$$

$$\therefore \frac{dy}{dx} = \frac{3e^{3x}}{2\sqrt{t}} = \frac{3e^{3x}}{2\sqrt{e^{3x}+1}}$$

when $x = \ln(2)$ $\frac{dy}{dx} = \frac{3e^{3\ln(2)}}{2\sqrt{e^{3\ln(2)}+1}}$

$$= \frac{3 \times 8}{2 \times \sqrt{9}} = 4$$

b)

x	y
0.25	1.7655
0.75	3.2385
1.25	6.5471
1.75	13.841

$$h = 0.5$$

$$\text{Area} = 0.5 \times 25.4421$$

$$= 12.7210$$

$$= 12.7 \text{ (3sf)}$$

c) Volume = $\pi \int y^2 dx$

$$y = (e^{3x} + 1)^{1/2}$$

$$\therefore y^2 = e^{3x} + 1$$

$$= \pi \int e^{3x} + 1 dx$$

$$= \pi \left[\frac{1}{3} e^{3x} + x \right]_0^2$$

$$= \pi \left[\frac{1}{3} e^6 + 2 - \frac{1}{3} e^0 - 0 \right]$$

$$= \pi \left[\frac{1}{3} e^6 + \frac{5}{3} \right] = \frac{\pi}{3} [e^6 + 5]$$

7 a) $y = \frac{\sin \theta}{\cos \theta}$

$u = \sin \theta$

$v =$

$\frac{du}{d\theta} = \cos \theta$

$\frac{dv}{d\theta} =$

$\frac{dy}{d\theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} = \sec^2 \theta$

b) $x = \sin(\theta)$

$x^2 = \sin^2(\theta)$

$\sin^2 \theta + \cos^2 \theta = 1$

$x^2 = 1 - \cos^2(\theta)$

$\cos^2(\theta) = 1 - x^2$

$\rightarrow \cos(\theta) = \sqrt{1 - x^2}$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{x}{\sqrt{1 - x^2}}$

c) $\int \frac{1}{(1 - x^2)^{3/2}} dx$

$x = \sin(\theta)$

$\frac{dx}{d\theta} = \cos(\theta)$

$dx = \cos(\theta) d\theta$

$= \int \frac{1}{(1 - \sin^2(\theta))^{3/2}} \cos \theta d\theta$

$1 - \sin^2 \theta = \cos^2 \theta$

$= \int \frac{1}{(\cos^2 \theta)^{3/2}} \cos \theta d\theta$

$= \int \frac{1}{(\cos^3 \theta)} \cos \theta d\theta$

$= \int \frac{1}{\cos^2 \theta} d\theta = \int \sec^2 \theta d\theta$

$= \tan(\theta)$

$= \frac{x}{\sqrt{1 - x^2}}$